

ECON0106: Microeconomics

Problem Set 9

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Question 1. There are two countries choosing fiscal policies, which determine the tax burden at each income quantile, that is, $s_i : [0, 1] \rightarrow [0, 1]$; we write $S_i := \{s_i : [0, 1] \rightarrow [0, 1]\}$ and, for any $s'_i, s_i \in S_i$, $s'_i \geq s_i$ iff $s'_i(q) \geq s_i(q)$, $\forall q \in [0, 1]$.

Each country i observes its current market confidence levels, summarised by Θ_i , where we write $\theta'_i \geq \theta_i$ to say that market confidence is higher under θ'_i than under θ_i .

Each country tries to improve its output level, which depends on both countries' tax policies and the own country's market confidence level, as given by $(s_i, s_{-i}, \theta_i) \mapsto u_i(s_i, s_{-i}, \theta_i) \in \mathbb{R}$.

Assume throughout that u_i is quasisupermodular in s_i , i.e., $u_i(s_i \vee s'_i, s_{-i}, \theta_i) \geq (>) u_i(s'_i, s_{-i}, \theta_i) \implies u_i(s_i, s_{-i}, \theta_i) \geq (>) u_i(s'_i \wedge s_i, s_{-i}, \theta_i)$.

(i) Interpret what quasisupermodularity in s_i of u_i means in this context.

(ii) Prove that $(S_i, \geq)$ is a complete lattice.

We make two assumptions. (1) Whenever lowering taxes decreases output for country i , then this output loss is lower when market confidence is higher or country j 's taxation level is lower. (2) In contrast, whenever lowering taxes increases output for country i , then it would lead to an even greater output increase when market confidence is higher or country j 's taxation level is lower.

(iii) Show that, under the above assumptions and regardless of the economic conditions of the countries, there is a (pure strategy) Nash equilibrium in this game.

(iv) Now suppose that the market confidence of country i increases. What can we say about the equilibrium tax policy of country i ? And about country j 's tax policy?

Now consider the alternative assumptions: (1') Whenever lowering taxes decreases output for country i , then this output loss is lower when market confidence is higher or country j 's taxation level is higher. (2') In contrast, whenever lowering taxes increases output for country

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i , then it would lead to an even greater output increase when market confidence is higher or country j 's taxation level is higher.

Further assume that u_i is continuous in (s_i, s_{-i}) with respect to the sup-norm, $\|\cdot\|_\infty$.

(v) Show that there is a (pure strategy) Nash equilibrium in this game.

(vi) Again, suppose that the market confidence of country i increases. What can we say about the equilibrium tax policy of country i ? And about country j 's tax policy?

Question 2. Player S , the seller, owns a good that is worth v to player B , the buyer. The value v is drawn from a commonly known distribution F and is private information of the buyer. The players bargain over the price of the good over two periods, with the seller making a take-it-or-leave-it offer (proposing a price) at the start of the first period, that the buyer can accept or reject. The game ends if an offer is accepted or after the two periods, whichever comes first. Both players discount period 2 payoffs with a discount factor of $\delta \in (0, 1)$. Assume for simplicity that the buyer always accepts an offer when indifferent.

(i) Characterise all the pure strategy weak perfect Bayesian equilibria of this game when the distribution of v is such that $\mathbb{P}(v = v_H) = 1 - \mathbb{P}(v = v_L) = q \in (0, 1)$, where $v_H > v_L > 0$.

(ii) Characterise all the pure strategy weak perfect Bayesian equilibria of this game when v is uniformly distributed on $[\underline{v}, \bar{v}]$.

Question 3. This question serves the purpose of reviewing and practicing the definitions of several of the solution concepts and refinements that we covered throughout the term via a numerical example (getting the definitions right is crucial though).

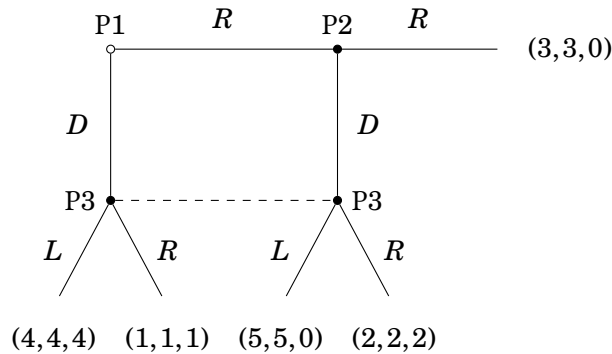


Figure 1:

Refer to the extensive-form game depicted in Figure 1.¹ Show your work; you need to prove your claims.

¹The payoffs for any given outcome x are given in parentheses $(u_1(x), u_2(x), u_3(x))$, where $u_i(x)$ refers to the payoff of player P_i , $i = 1, 2, 3$. Note that P1 chooses at the root of the game.

- (i) Describe the normal-form of the game presented above, that is, the set of players, pure strategies for each player, and payoffs.
- (ii) Find *all* Nash equilibria (both pure and mixed).
- (iii) Find *all* Trembling-Hand Perfect Equilibria.
- (iv) Find *all* Subgame-Perfect Nash Equilibria.
- (v) Find *all* Weak Perfect Bayesian Equilibria.
- (vi) Find *all* Sequential Equilibria.